Exercises - Part 2[1]

For Chapters 06 to 10

See Exercise Sections in
Reinhard Klette: Concise Computer Vision
Springer-Verlag, London, 2014

[1]See last slide for copyright information.

Agenda

6.1–6.2

Exercise

Check lectures to Chapter 6 for equations given in inhomogeneous coordinates. Express all those in homogeneous coordinates.

Exercise

Specify the point at infinity on the line $31x + 5y - 12 = 0$.
Determine the homogeneous equation of this line. What is the intersection point of this line with the line $31x + 5y - 14 = 0$ *at infinity?*
Generalize by studying lines $ax + by + c_1 = 0$ *and* $ax + by + c_2 = 0$, *for* $c_1 \neq c_2$.

6.3

Exercise

Consider a camera defined by the following 3×4 *camera matrix:*

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Compute the projections of the following 3D points (in world coordinates) with this camera:

$$\mathbf{P}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{P}_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \mathbf{P}_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \text{ and } \mathbf{P}_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

6.4

Exercise

Let $p_R = [x, y, 1]^\top$ *and* $p_L = [x', y', 1]^\top$. *Equation* $p_R^T \cdot \mathbf{F} \cdot p_L = 0$ *is equivalently expressed by*

$$\begin{bmatrix} xx' & xy' & x & yx' & yy' & y & x' & y' & 1 \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{21} \\ F_{31} \\ F_{12} \\ F_{22} \\ F_{32} \\ F_{13} \\ F_{23} \\ F_{33} \end{bmatrix} = 0$$

where F_{ij} *are the elements of the fundamental matrix* $\mathbf{F}$. *Now assume that we have at least 8 pairs of corresponding pixels, defining the matrix equation*

6.4 Continued

Exercise

$$\begin{bmatrix} x_1x_1' & x_1y_1' & x_1 & y_1x_1' & y_1y_1' & y_1 & x_1' & y_1' & 1 \\ x_2x_2' & x_2y_2' & x_2 & y_2x_2' & y_2y_2' & y_2 & x_2' & y_2' & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_nx_n' & x_ny_n' & x_n & y_nx_n' & y_ny_n' & y_n & x_n' & y_n' & 1 \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{21} \\ \vdots \\ F_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

for $n \geq 8$, expressed in short as

$$\mathbf{A} \cdot \mathbf{f} = \mathbf{0}$$

Solve this equation for the unknowns F_{ij}, considering noise or inaccuracies in pairs of corresponding pixels.

6.5

Exercise

Show that the following is true for any nonzero vector $\mathbf{t} \in \mathbb{R}^3$*:*

1. $[\mathbf{t}]_x \cdot \mathbf{t} = 0$,
2. *the rank of matrix* $[\mathbf{t}]_x$ *is 2,*
3. *the rank of the essential matrix* $\mathbf{E} = \mathbf{R}[\mathbf{t}]_x$ *is two,*
4. *the fundamental matrix* $\mathbf{F}$ *is derived from the essential matrix* $\mathbf{E}$ *by the formula* $\mathbf{F} = \mathbf{K}_R^{-T} \mathbf{E} \mathbf{K}_L^{-1}$,
5. *the rank of the fundamental matrix* $\mathbf{F}$ *is two.*

Agenda

7.1

Exercise

A smooth compact *3D set is compact (i.e., connected, bounded and topologically closed) and curvature is defined at any point of its frontier (i.e., its surface is differentiable at any point).*
Prove (mathematically) that the similarity curvature measure

$$\mathcal{S}(P) = \begin{cases} (\kappa_3, 0) & \text{if signs of } \kappa_1 \text{ and } \kappa_2 \text{ are both positive} \\ (-\kappa_3, 0) & \text{if signs of } \kappa_1 \text{ and } \kappa_2 \text{ are both negative} \\ (0, \kappa_3) & \text{if signs of } \kappa_1 \text{ and } \kappa_2 \text{ differ, and } |\kappa_2| \geq |\kappa_1| \\ (0, -\kappa_3) & \text{if signs of } \kappa_1 \text{ and } \kappa_2 \text{ differ, and } |\kappa_1| > |\kappa_2| \end{cases}$$

is scaling invariant, for any smooth compact 3D set.

7.2

Exercise

Instead of applying a trigonometric approach for structured lighting, specify the required details for implementing a linear-algebra approach for structured lighting along the following steps:

1. *From calibration we know the implicit equation for each light plane expressed in world coordinates.*
2. *From calibration we also know in world coordinates the parametric equation for each ray from the camera's projection center to the center of a square pixel, for each pixel.*
3. *Image analysis gives us the ID of the light plane visible at a given pixel location.*
4. *An intersection of the ray with the light plane gives us the surface-point coordinates.*

7.3–7.5

Exercise

Specify the fundamental matrix **F** *for canonical stereo geometry. Consider also a pair of tilted cameras as shown in Figure* **??***, and specify also the fundamental matrix* **F** *for such a pair of two cameras.*

Exercise

Describe the Lambertian reflectance map in spherical coordinates on the Gaussian sphere (hint: isointensity curves are circles in this case). Use this model to answer the question why two light sources are not yet sufficient for identifying a surface normal uniquely.

Exercise

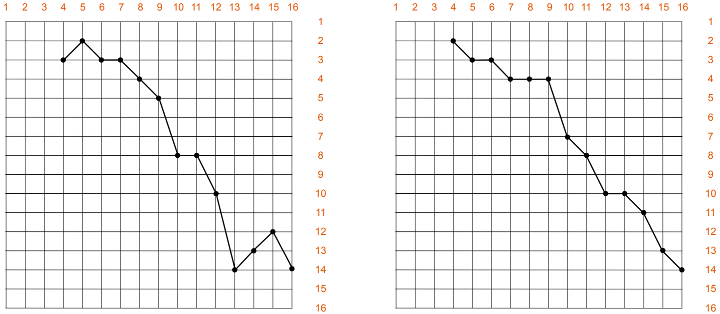
Why the statement "We also use Parseval's theorem..." when introducing the gradient-field integration algorithms (with deriving solutions in frequency space)?

Agenda

8.1

Exercise

Assume that our stereo system needs to analyze objects being in a distance of at least a *meters from our binocular stereo camera system. We have intrinsic and extrinsic camera parameters calibrated. Determine value* d_{max} *based on those parameters and known value* a.



8.2

Exercise

The figure on the page before shows two profiles. Are these epipolar profiles which represent disparity vectors $\mathbf{d}$*? If so, which disparity vector? In reverse: Given are the disparity vectors*

$$\mathbf{d}_1 = [f_4, f_5, \ldots, f_{16}]^\top = [1, 0, 1, 2, 3, 0, 1, 1, 0, 1, 2, 3, 2]^\top$$

$$\mathbf{d}_2 = [f_4, f_5, \ldots, f_{16}]^\top = [4, 3, 2, 1, 0, 1, 2, 4, 1, 2, 3, 2, 2]^\top$$

Draw the epipolar profiles defined by those two disparity vectors. Which profiles and which vectors satisfy the ordering constraint?

8.3

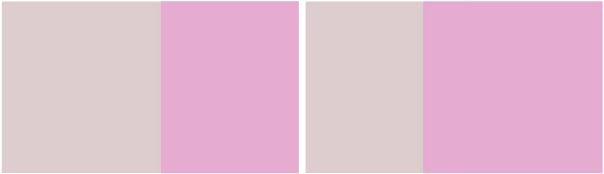
Exercise

In the lectures we discussed how 4-adjacency "grows" into the image carrier by repeated creations of dependencies between adjacent pixels. At time $t = 0$ it is just the pixel itself ($n_0 = 1$), at time $t = 1$ also the four four-adjacent pixels ($n_1 = n_0 + 4 = 5$), at time $t = 2$ also eight more pixels ($n_2 = n_1 + 8 = 13$).

How many pixels are in this growing set at time $t \geq 0$ in general, assuming no limitation by image borders. At the time τ when terminating the iteration, n_τ defines the cardinality of the area of influence.

Now replace 4-adjacency by 8-adjacency and do the same calculations. As a third option, consider 4-adjacency but also a regular image pyramid "on top" of the given image.

8.4



Exercise

Stereo matchers have to work on any input pair? Fine, here is one - see the figure above.

Assume the simple AD data cost function and discuss (as a "Gedanken experiment") outcomes of "The winner takes all", DPM with ordering constraint, multi-scanline DPM with smoothness constraint, and of BPM for this stereo pair.

Agenda

9.1–9.2

Exercise

In the lecture we discussed a proposal how to detect keypoints at subpixel accuracy. Assume values a_N, a_E, a_S, a_W (for the 4-adjacent pixel locations) and a_p (at the detected keypoint pixel) for function g and provide a general solution for subpixel accuracy.

Exercise

The algorithm (provided in the lectures) for plane fitting to a sparse set of 3D points lists two procedures `ransacFitPlane` *and* `ransacRefinePlaneModel`*. Specify such two procedures for initial and refined plane fitting following the general RANSAC idea.*

9.3–9.5

Exercise

For lane-border detection it was suggested in the lectures to generate a bird's-eye view by using a homography, defined by four marked points being corners of a trapezoid in the image, but actually corners of a rectangular region on the road. Specify this homography.

Exercise

Explain the motivations behind the definitions of particle weights given in the lectures for the presented lane-border detection approach.

Exercise

Show that $\mathbf{F}_{\Delta t} = \mathbf{I} + \Delta t \mathbf{A} + \frac{\Delta t^2}{2} \mathbf{A}^2$ *for matrix* $\mathbf{A}$ *as defined in the first example in the Kalman filter section.*

Agenda

10.1–10.2

Exercise

Continue the calculations (at least one more iteration step) for the given AdaBoost example.

Exercise

Do manually AdaBoost iterations for six descriptors $\mathbf{x}_1$ *to* $\mathbf{x}_6$ *when having three weak classifiers (i.e.* $w = 3$*), denoted by* h_1*,* h_2*, and* h_3*, where* h_1 *assigns class number "+1" to any of the six descriptors, classifier* h_2 *assigns class number "-1" to any of the six descriptors, and classifier* h_3 *assigns class number "+1" to* $\mathbf{x}_1$ *to* $\mathbf{x}_3$*, and class number "-1" to* $\mathbf{x}_4$ *to* $\mathbf{x}_6$*.*

10.3

Exercise

Let $S = \{1, 2, 3, 4, 5, 6\}$*,* X *and* Y *are random variables defined on* S*, with* $X = 1$ *if the number is even, and* $Y = 1$ *if the number is prime (i.e. 2, 3, or 5). Let the values of joint or conditional probabilities* $p(x, y)$ *and* $p(y|x)$ *be defined as follows:*

$$\begin{aligned} p(x, y) &= P(X = x, Y = y) \\ p(y|x) &= P(X = x|Y = y) \end{aligned}$$

Give values for all possible combinations, such as for $p(0, 0)$ *or* $p(0|1)$*.*

10.4

Exercise

Consider a finite alphabet $S = \{a_1, \ldots, a_m\}$ and two different random variables X and Y taking values from S, with $p_j = P(X = a_j)$ and $q_j = P(Y = a_j)$. The relative entropy of discrete probability p with respect to discrete probability q is then defined as

$$H(p|q) = -\sum_{j=1}^{m} p_j \cdot \log_2 \frac{p_j}{q_j}$$

Show that

1. $H(p|q) \geq 0$
2. *There are cases where $H(p|q) \neq H(q|p$.*
3. *$H(p|q) = 0$ iff $p = q$ (i.e. $p_j = q_j$, for $j = 1, \ldots, m$).*

10.5–10.6

Exercise

Let X be a random variable that takes values in the alphabet $S = \{A, B, C, D, E\}$.
Calculate the Huffman codes (not explained in this book; check other sources if needed) for the following two probability distributions:

1. *Let X takes values A, B, C, D, or E with uniform probability $1/5$.*
2. *Now consider probabilities $P(X = A) = 1/2$, $P(X = B) = 1/4$, $P(X = C) = 1/8$, $P(X = D) = 1/16$, and $P(X = E) = 1/16$.*

Exercise

Verify that $H(Y|c) = 0$ in the given example for illustrating conditional entropy.

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