Abbreviations

2D	2-dimensional
3D	3-dimensional
AMLPP	Approximate minimum-length pseudo-polygon; see Page 273
CAV	Cavity; see Page 102
CH	Convex hull; see Page 101
ESP	Euclidean shortest path; see Page 19
iff	Read: if and only if; see Page 14
MLA	Minimum-length arc; see Page 276
MLP	Minimum-length polygon; see Page 118, 229–231, 301
MLPP	Minimum-length pseudo-polygon; see Page 273
MPP	Minimum-perimeter polygon; see Page 115
RBA	Rubberband algorithm; see Page 63
RCH	Relative convex hull; see Page 120
SPM	Shortest path map; see Page 161
SRP	Safari route problem; see Page 345
TPP	Touring polygon problem; see Page 309
WRP	Watchman route problem; see Page 323
ZRP	Zookeeper route problem; see Page 345

Symbols

$\|S\|$	Cardinality of set S
$\parallel$	Relation sign for being parallel
∂S	Frontier of set S
$\wedge$	Logical 'and'
$\vee$	Logical 'or'
$\cap$	Intersection of sets
$\cup$	Union of sets
pq	Straight line segment with endpoints p and q
pqr	Triangle with vertices p, q, and r
$\triangle$	Trapezoid or triangle (in a partitioning)
$\overline{pq}$	Straight line defined by points p and q; orientation "from p to q"
$\sphericalangle(pqr)$	Angle formed by rotating segment pq clockwise into segment qr
$\square$	End of a proof or of an example
a,b,c	Real numbers
$\mathscr{A}(.)$	Area of a measurable set (as a function)
α,β	Angles
$\mathbb{C}$	Set of complex numbers $a+i\cdot b$, with $i=\sqrt{-1}$ and $a,b\in\mathbb{R}$
d_m	Minkowski metrics, for $m=1,2,\ldots$ or $m=\infty$
d_e	Euclidean metric; note that $d_e=d_2$
D	Determinant; see Page 100
δ	Real number greater than zero
e	Edge (e.g. of a graph); real constant $e=\exp(1)\approx 2{\cdot}7182818284$
E	Set of edges of a graph
ε	Real number greater than zero (the accuracy of an RBA)
ε_s	Real number greater than zero (a shift distance)
f,g,h	Functions, for example from $\mathbb{N}$ into $\mathbb{R}$
g	Simple cube-curve
$\mathbf{g}$	Tube (i.e., the union of cubes) of a simple cube-curve g
F	Face of a polyhedron

$\mathscr{F}$	Set of faces of a polyhedron
G	Graph
$\mathbb{G}$	Set $\{\ldots,-2,-1,0,1,2,\ldots\}$ of integers
γ	Curve in Euclidean space (e.g. straight line, polyline, smooth curve)
H	Half plane; see Page 99
i,j,k,l,m,n	Natural numbers
i	Number of iterations, e.g. in a rubberband algorithm
j	Natural number; index of points or vertices in a path
k	Natural number; total number of items
κ	Function in ε
L	Length (as a real number)
$\mathscr{L}(.)$	Length of a rectifiable curve (as a function)
$l(\rho)$	Length of a path ρ
λ	Real number; e.g. between 0 and 1
n	Natural number; e.g. defining the complexity of the input
N	Neighbourhood (in the Euclidean topology, or in grids)
$\mathbb{N}$	Set $\{0,1,2,\ldots\}$ of natural numbers
$\mathscr{O}(.)$	Asymptotic upper time bound
p,q	Points in $\mathbb{R}^2$ or $\mathbb{R}^3$, with coordinates x_p, y_p, or z_p
$p(x)$	Polynomial in x
$p.x, p_i.x$	x-coordinate of point p or point p_i
$p.y, p_i.y$	y-coordinate of point p or point p_i
$p.z, p_i.z$	z-coordinate of point p or point p_i
P	Polygon
$P^\bullet$, P°	Closure and topological interior of polygon P
π	Plane in $\mathbb{R}^3$; real constant $\pi = 4 \times \arctan(1) \approx 3{\cdot}14159265358979$
Π	Polyhedron
$\Pi^\bullet$, Π°	Closure and topological interior of polyhedron Π
$\Pi_{i=1}^{k} S_i$	Product of sets S_i
$\mathscr{P}$	Partitioning (of the plane or of a simple polygon)
r	Radius of a disk or sphere; point in $\mathbb{R}^2$ or $\mathbb{R}^3$
$\mathbb{R}$	Set of real numbers
ρ	Path with finite number of vertices; see Page 19
s	Point in $\mathbb{R}^2$ or $\mathbb{R}^3$
S, S_i	Sets
$\mathscr{S}$	Family of sets
t	Point in $\mathbb{R}^2$ or $\mathbb{R}^3$
T	Tree; threshold (a real number)
τ	Threshold (real number)
$\mathscr{T}$	Trapezoidation or triangulation (of the plane or of a simple polygon)

v	Vertex or node; a point in $\mathbb{R}^2$ or $\mathbb{R}^3$, with coordinates x_v, y_v, or z_v
V	Set of vertices of a graph
$V(G), V(T)$	Set of vertices of graph G or tree T
$V(\rho)$	Set of vertices of a path ρ
$vu, \overrightarrow{vu}$	Undirected or directed line segment between points v and u
x	Real variable
$\mathbf{x}$	Vector
y	Real variable
$\mathbf{y}$	Vector
$\mathbb{Z}$	Set of integers

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