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Animal and B-Problems

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(1) The animal problem: This is a fairly well-known open problem; see page 276 of [1]. An animal — according to Janos Pach— is any topological 3-ball in R^3, consisting of unit cubes. In general, we can define an animal in R^n (where n is a positive integer) that is called an *n-animal*. The question called *animal problem* is whether every 3-animal can be reduced to a single unit cube by a finite sequence of either adding or removing a cube, while maintaining the animal property throughout. It is not so difficult to solve the 2-animal problem, and there are various methods to solve this problem. For example, it is sufficient to use the magnification technique as published in my paper inf the proceeding (LNCS 3322) of IWCIA04. However, this method is not applicable to the 3-animal problem since there is a local pattern in an animal A such that we can upward dilate A by SD (= simple deformation) but not deform A by preserving animality. An example of such a local pattern is the following:

$$\{(0,0,0),(0,0,1),(1,0,0),(1,1,0),(2,1,0),(2,1,1)\}$$

The difficulty lies in "animality-preserving". However, if we permit (any) SD instead of animality- preserving transforms only, the problem seems to be solvable. When I was collaborating with the late Prof. A. Rosenfeld, he called this the *B-problem*. We state it as follows:

(2) B-problem: Let B be a connected component in a 3D digital picture $P = (Z^3, 26, 6, B)$ such that B does not contain any cavity or any tunnel. Is B SD-equivalent to a single voxel?

This problem may be easier solvable than the animal problem. As for this problem, I have a proof idea, and I am now trying to formulate a strict proof.

(3) The n-dimenional animal problem: This is the extension of (1) to the nD case.

(4) The n-dimensional B-problem: This is the extension of (2) to the nD case.

References

[1] G. M. Ziegler: *Lectures on Polytopes*. Springer, New York, 1995.